

## Nash optimality based distributed model predictive control for vehicle platoon<sup>\*</sup>

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**Abstract:** In this paper, a distributed model predictive control algorithm (DMPC) based on Nash optimality is proposed for automated vehicle platoon control. The optimization decision of vehicle platoon is decomposed into the decentralized optimization of single vehicles, in which the Nash optimality algorithm is adopted to solve the decentralized optimization problem. Thus, each vehicle can reach the local optimal target and the whole team can reach its Nash equilibrium. The methodology employs neighborhood information of the entire platoon through on-board sensors and V2V communication to achieve coordination of the entire platoon. The ability of the methods in terms of robustness to disturbances and cyber-physical interaction is demonstrated with simulation case studies.

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**Keywords:** Vehicle platoon, Distributed control, Model predictive control, Nash optimality.

### 1. INTRODUCTION

A platoon (or road train) is a collection of vehicles where a manually driven heavy lead vehicle is followed by several automatically controlled following trucks or passenger cars. The platooning of autonomous vehicles has potential to improve traffic efficiency, enhance road safety and reduce fuel consumption (Stankovic et al., 2000; Ploeg et al., 2014; Zheng et al., 2017). The vehicle platoon should bring the vehicle distance towards a prescribed vehicle distance and velocity in stationary situations, and avoid collisions for arbitrary behaviour of the leading vehicle.

Distributed control has dominated vehicle platoons due to its reliable control structure, good adaptability and robustness (Godbole and Lygeros, 1993; Stotsky et al., 1995; Gao et al., 2018; Feng et al., 2019), where a suitable communication topology and vehicle spacing strategy must be adopted. Typical communication topology of vehicle platoons mainly includes predecessor-following topology, predecessor-leader following topology, predecessors-following topology, two-leader following topology (Zheng et al., 2015; Chen et al., 2018; Li et al., 2019). Communication topologies have an important impact on the platoon performance and collision avoidance (Seiler et al., 2004; Bernardo et al., 2015). The car spacing strategy can be classified into fixed car spacing, fixed headway

distance and variable headway distance (Yanakiev and Kanellakopoulos, 1995, 1998; Ali et al., 2015; Ma et al., 2017), where the variable headway ensures the stability of the traffic flow and the platoon. Model predictive control (MPC) provides an effective way to control a large and practical class of nonlinear multi-input multi-output systems, and deal with constraints in a straightforward way (Rawlings et al., 2017; Rakovic and Levine, 2018). Distributed model predictive control (DMPC) has been intensively discussed in the last decades, and many DMPC schemes are nowadays available (Christofides et al., 2013; Negenborn and Maestre, 2014; Maestre and Negenborn, 2014; Rawlings et al., 2017; Trodden and Maestre, 2017; Rakovic and Levine, 2018). Recently, DMPC schemes are presented for vehicle platoons in which each vehicle solves its own optimization problem, determines its own control over the prediction horizon, and transmits and receives information with its assigned neighbour (Dullerud and Caveney, 2012; Zheng et al., 2013, 2015, 2017). An iterative DMPC scheme is developed based on Nash optimality for large-scale systems to tackle the state coupling between subsystems, and the relevant computation convergence and the nominal stability condition are presented (Li et al., 2005b; Giovanini and Balderud, 2011; Zhang et al., 2015).

Since the protocol of mutual communication and information exchange is adequately taken into account, Nash optimality based DMPC can efficiently improve control performance. In this paper a Nash optimality based model predictive control of vehicle platoons is proposed, where the

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distributed controllers can exchange information several times during each optimization process. Although the optimization problems of DMPC are solved only with locally relevant variables, costs and constraints, some degree of coordinate among vehicles are achieved. Safety or collision avoidance is formulated as the time-domain constraints in the optimization problem of the Nash optimality based model predictive control.

The remainder of this paper is organized as follows: Section 2 introduces the problem of platoon control, including vehicular longitudinal dynamics, predecessors-following topology and platoon model. A distributed model predictive control algorithm based on Nash optimality is proposed in Section 3. Section 4 illustrates the effectiveness of the adopted methodology with a four-vehicle platoon control example. Section 5 is for concluding remarks.

## 2. MODELING DESCRIPTION AND PROBLEM FORMULATION

This section starts with the introduction of the single vehicle model and follows with the communication topology among vehicles and platoon model. All vehicles in the platoon maintain a cooperative relationship: each of them needs to meet related constraints and to ensure the safety and consistency of the entire platoon.

As long as the vehicles in the platoon are able to follow the corresponding vehicles at the desired speed and spacing policy, the whole platoon can drive steadily in the desired formation and speed.

Note that this paper focuses on the longitudinal control of a platoon, i.e., the whole platoon moves along the same straight lane.

### 2.1 Vehicle dynamics

Suppose that the  $i$ th automotive vehicle in a platoon can be represented by the following nonlinear third-order model (Zhang, 2011)

$$\begin{cases} \dot{s}_i = v_i \\ \dot{v}_i = a_i \\ \dot{a}_i = f_i(v_i, a_i) + g_i(v_i)\eta_i \end{cases} \quad (1)$$

where  $s_i$  is the position of the vehicle,  $v_i$  and  $a_i$  are the speed and the acceleration of the vehicle, respectively, and  $\eta_i$  is the engine input. Eq.(1) represents the longitudinal dynamics of a vehicle.

The functions  $f$  and  $g$  can be written as

$$f_i(v_i, a_i) = \frac{-2C_{di}}{m_i}v_ia_i - \frac{1}{\tau_i(v_i)} \left[ a_i + \frac{C_{di}}{m_i}v_i^2 + \frac{d_{mi}}{m_i} \right]$$

$$g_i(v_i) = \frac{1}{m_i\tau_i(v_i)}$$

where  $C_{di}$  is the aerodynamic drag coefficient,  $m_i$  the vehicle mass,  $\tau_i$  the time constants of its engine,  $d_m$  the mechanical drag (Stankovic et al., 2000).

Suppose that the parameters of Eq.(1) are *a priori* known, and the  $\tau_i$  is constant. Choosing a nonlinear control law of the nonlinear system (1) as

$$\eta_i = mu_i + C_{di}v_i^2 + d_{mi} + 2\tau_iC_{di}v_ia_i, \quad (2)$$

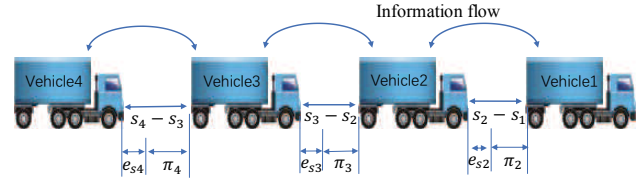


Figure 1. Predecessors-following topology

then the nonlinear system (1) can be transformed into the equivalent linear system

$$\begin{cases} \dot{s}_i = v_i \\ \dot{v}_i = a_i \\ \dot{a}_i = -\tau_i^{-1}a_i + \tau_i^{-1}u_i \end{cases} \quad (3)$$

in which the control input  $u_i$  can be treated as the expected acceleration of the vehicle. Note that the control law  $\eta_i$  achieves feedback linearization.

For simplicity, three assumptions are made in this paper.

**Assumption 1.** Only homogeneous vehicle platoon is considered in this paper, i.e.,  $\tau_i = \tau$ ,  $d_i = d$  and  $h_i = h$ .

**Assumption 2.** The vehicle status  $s_i$ ,  $v_i$  and  $a_i$  can be measured instantaneously for all  $i \geq 1$ .

**Assumption 3.** The clock of each vehicle is synchronized, i.e., to behave simultaneous or near-simultaneous from a certain perspective.

### 2.2 Communication topology

Vehicle to vehicle (V2V) communication system is a short-range communication technology that enables vehicles to share data such as vehicle speed. The communication topology determines the information interaction between the member vehicles in the vehicle platoon and expands the scope of the environment perception of vehicles.

Vehicle platoon control usually uses a distributed controller, which calculates the control law based only on limited neighbor status information. The topology determines the car-following strategy of the member vehicles in the vehicle platoon (Li et al., 2005a) as well.

This paper adopts the predecessors-following topology shown (four-vehicle platoon) in Fig.1. Under this communication topology, the vehicle can obtain the current information from the preceding vehicle in front of it.

**Remark 1.** The acceleration of the leading vehicle, Vehicle 1, in a time interval is known *a priori* with the technology of pattern recognition and decision making accordingly.

### 2.3 Platoon modeling

The most important thing for a vehicle platoon is to maintain a small safe distance between the vehicles through the given inter vehicle communication. Safe distance is the distance between the vehicle and the vehicle in front of it. The smaller the safe distance, the better the road traffic capacity and the efficiency of the platoon.

The safety distance of a vehicle platoon refers to the time interval between two consecutive vehicles passing through the same position

$$\pi_i = d_i + h_iv_i \quad (4)$$

where the constant  $d_i$  is the minimum safe distance, the constant  $h_i$  the headway,  $v_i$  the current vehicle speed. As the current speed increases, so does the safe distance between vehicles.

For all  $i \geq 2$ , define

$$e_{si} = s_{i-1} - (s_i + \pi_i)$$

and

$$e_{vi} = v_{i-1} - v_i$$

where  $e_{si}$  and  $e_{vi}$  represent the distance difference and speed difference between  $i$  vehicle and  $i - 1$  vehicle in the platoon.

Define  $x_i := [e_{si} \ e_{vi} \ a_i]^T$ , then

$$\dot{x}_i = \hat{A}_i x_i + \hat{B}_{2i} u_i + \hat{B}_{1i} a_{i-1} \quad (5)$$

$$\text{with } \hat{A}_i = \begin{bmatrix} 0 & 1 & -h \\ 0 & 0 & -1 \\ 0 & 0 & -\tau^{-1} \end{bmatrix}, \hat{B}_{2i} = \begin{bmatrix} 0 \\ 0 \\ \tau^{-1} \end{bmatrix}, \hat{B}_{1i} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Denote the sampling time as  $\delta \geq 0$ . Suppose that the first-order retainer is adopted, then the discrete-time system related to the continuous-time system (5) is

$$x_i(k+1) = A_i x_i(k) + B_{2i} u_i(k) + B_{1i} a_{i-1}(k) \quad (6)$$

Note that the system (6) is controllable.

**Remark 2.** The preceding acceleration of  $a_{i-1}$  is kind of “reference” for all Vehicle  $i \geq 2$  since the prediction or estimation value of  $a_{i-1}$  has already known by communication.

## 2.4 Objective of vehicle platoon control

The goal of a vehicle platoon is to track the speed of the leader

$$\lim_{k \rightarrow \infty} v_i(k) - v_1(k) = 0, \quad i \geq 2, k \geq 0, \quad (7)$$

while maintaining a desired gap between any consecutive vehicles which is specified by the desired spacing policy, and the safety distance

$$0 \leq s_{i-1}(k) - s_i(k) - D_i \leq e_{\max}, \quad i \geq 2, k \geq 0. \quad (8)$$

where  $e_{\max}$  reflects the maximum permissible distance from the car in front. Note that safety or collision avoidance constraints (8) are required to guarantee that the vehicle with proposed controller always keeps a safe distance from the preceding vehicle.

In order to avoid large deviation from the desired speed, the relative speed between the two adjacent vehicles has to be constrained

$$e_{vi,\min} \leq e_{vi}(k) \leq e_{vi,\max}, \quad i \geq 2, k \geq 0. \quad (9)$$

where  $e_{vi,\min}$  and  $e_{vi,\max}$  are the allowed minimum and maximum speed deviation, respectively.

To quantify road jerks, the longitudinal acceleration of the vehicle has to be kept within an acceptable range

$$a_{i,\min} \leq a_i(k) \leq a_{i,\max}, \quad i \geq 2, k \geq 0. \quad (10)$$

where  $a_{i,\min}$  and  $a_{i,\max}$  are the allowed minimum and maximum acceleration, respectively.

## 3. DISTRIBUTED MODEL PREDICTIVE CONTROL

The distributed model predictive control breaks the whole system up into individual subsystems which communicate

only with their chosen neighbours, and solve their own optimization problem in parallel. The individual feasible solution is applied to the subsystem and the updated information is transformed to its neighbor accordingly.

For each Vehicle  $i$  with  $i \geq 2$ , define the sequence of the control input at the time instant  $k$  as follows

$$U_i(k) := \{u_i(k|k), u_i(k+1|k), \dots, u_i(k+N-1|k)\}. \quad (11)$$

Then, the open-loop optimization problem of the distributed model predictive control is formulated accordingly as follows:

### Problem 1.

$$\text{minimize}_{U_i(k)} \quad J_i(x_i(k), U_i(k))$$

subject to:

$$x_i(k+j+1|k) = A_i x_i(k+j|k) + B_{2i} u_i(k+j|k) \quad (12a)$$

$$+ B_{1i} \bar{a}_{i-1}(k+j|k) \quad (12b)$$

$$x_i(k|k) = x_i(k)$$

$$s_{i-1}(k+j+1|k) - s_i(k+j|k) - D_i \in [0, e_{\max}] \quad (12c)$$

$$e_{vi}(k+j|k) \in [e_{vi,\min}, e_{vi,\max}] \quad (12d)$$

$$a_i(k+j|k) \in [a_{i,\min}, a_{i,\max}] \quad (12e)$$

where  $\tilde{x}_i := [e_{s,i} \ e_{v,i} \ a_i - \bar{a}_i]^T$ ,

$$J_i(x_i(k), U_i(k)) = \sum_{j=0}^{N-1} \|\tilde{x}_i(k+j|k)\|_Q^2 + \|u_i(k+j|k)\|_R^2 \quad (13)$$

$N$  is the prediction horizon, and  $Q$  and  $R$  are positive definite matrices. Note that  $k+i|k$  is the predicted value at the time instant  $k+i$  starting from the time instant  $k$ .

The term  $\bar{a}_{i-1}(k+j|k)$  with  $j \in [0, N-1]$  is the prediction of the future acceleration of the preceding vehicle  $i$ , which will be introduced later.

Denote  $U_i^*(k) := \{u_i^*(k|k), u_i^*(k+1|k), \dots, u_i^*(k+N-1|k)\}$  and  $J_i^*(x_i(k)) := J_i(x_i(k), U_i^*(k))$  as the “optimal” control sequence and the related “optimal” cost function. For MPC, only the first element of  $U_i^*(k)$ , i.e.,  $u_i^*(k|k)$ , is applied to the vehicle  $i$  at the time instant  $k$ . At the next time instant, the whole process is repeated with the new measurement and information exchange.

### 3.1 Nash optimality

According to different types of optimization algorithms, the coordination method of DMPC can be categorized into non-iterative algorithm and iterative algorithm, respectively. Non-iterative algorithms allow each local controller exchanges information once per sample period with all other local controllers while its optimization problem is solved. On the contrary, iterative algorithms permit each local controller exchanges information multiple times with all other local controllers during each sampling period. Thus, iterative algorithms can achieve similar performance compared with centralized model predictive control with the price of a heavy communication and computation burden.

Nash optimality is an iterative algorithm which was proposed by Nash (1951) to solve the cooperative game problem. The control problem of vehicle platoon itself is a multi-vehicle cooperation problem, i.e., through the design

of each local controller and the negotiation between local controllers, the whole vehicle platoon can move in a given speed steadily. This subsection presents a Nash optimality based distributed model predictive control of vehicle platoon.

For each Vehicle  $i$  with  $i \geq 2$ , define the sequence of the prediction of the preceding vehicle acceleration at the time instant  $k$

$$\bar{A}_i(k) := \{\bar{a}_{i-1}(k|k), \bar{a}_{i-1}(k+1|k), \dots, \bar{a}_{i-1}(k+N-1|k)\}, \quad (14)$$

Note that the initial condition of  $\bar{a}_{i-1}(k+j|k)$  can be chosen as

$$\bar{a}_{i-1}(k+j|k) := \begin{cases} \bar{a}_{i-1}(k+j|k-1) & j \in [0, N-2] \\ 0 & j = N-1 \end{cases}$$

and then updated with the iteration.

For Vehicle  $i$  with  $i \geq 2$ , denote the  $h$ th iteration value at the time instant  $k$  of  $e_{si}(k)$ ,  $e_{vi}(k)$ ,  $a_i(k)$ ,  $\bar{A}_i(k)$ ,  $J_i(x_i(k), U_i(k))$  and  $U_i(k)$  as  $e_{si}^{\{h\}}(k)$ ,  $e_{vi}^{\{h\}}(k)$ ,  $a_i^{\{h\}}(k)$ ,  $\bar{A}_i^{\{h\}}(k)$ ,  $J_i^{\{h\}}(x_i(k), U_i^{\{h\}}(k))$  and  $U_i^{\{h\}}(k)$  accordingly.

Denote  $\tau > 0$  as the threshold of the Nash optimality based DMPC. For  $h \geq 2$ , if

$$|J_i^{\{h\}}(x_i(k), U_i^{\{h\}}(k)) - J_i^{\{h-1\}}(x_i(k), U_i^{\{h-1\}}(k))| \leq \tau, \quad (15)$$

i.e., the accuracy of the objective function of two consecutive times for any vehicle is satisfied, then  $U_i^*(k) := U_i^{\{h\}}(k)$ ,  $\bar{A}_i^*(k) := \bar{A}_i^{\{h\}}(k)$ ,  $J_i^*(x_i(k)) = J_i^{\{h\}}(x_i(k), U_i^{\{h\}}(k))$  and the iteration is terminated (Giovannini and Balderud, 2011).

Each vehicle, for example Vehicle  $i$ , will carry out the next iteration according to the latest state of the adjacent vehicles, and pass the obtained solution to the adjacent vehicles until the whole platoon converges to its Nash equilibrium point. There exists persistent communication between adjacent vehicles in the platoon which ensures the control and state information are shared while the corresponding optimization problem is solved online. For detail, please see Algorithm 1.

In terms of game theory, if each player has chosen a strategy, and no player can benefit by changing strategies while the other players keep theirs unchanged, then the current set of strategy choices and their corresponding payoffs constitutes a Nash equilibrium. If one vehicle in the platoon does not obtain its solution of Nash optimality (Nash equilibrium), then the other vehicle will also have to solve its own optimization problem accordingly in terms of the information exchange through communication with its adjacent vehicles. A solution of Nash optimality for Vehicle  $i$  is acceptable to all vehicles in the platoon.

If the algorithm is convergent (terminated), all the terminal conditions of the Vehicle  $i$  will be satisfied, and the whole system will arrive at its Nash equilibrium. This process will be repeated at the next sampling time.

**Remark 3.** Problem 1, solved at each time instant for each vehicle, is a convex optimization problem. Thus, there exists a unique solution to it which satisfies the given initial condition (Boyd and Vandenberghe, 2004).

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#### Algorithm 1 Nash optimality based DMPC

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**Input:** The threshold  $\tau$ , state variables  $e_{si}$ ,  $e_{vi}$ ,  $a_i$  and the prediction of the preceding vehicle acceleration  $\bar{A}_i$ ;

**Output:** Desirable acceleration  $u_i^*(k)$ ;

- 1: At time  $k$ , set  $h = 0$ ;
  - 2: Solve Problem 1 in parallel and passes the solution to its neighbour;
  - 3: Check whether all vehicles satisfies its iteration terminal condition:
    - If the iteration termination condition (15) is satisfied, then end the process of iteration, and take the current control input obtained  $u_i^{\{h\}}(k|k)$  as the solution and jumps to step 4;
    - If the iteration termination condition (15) is not satisfied, denote the prediction of the preceding acceleration  $\bar{A}_i^{\{h\}}$  as  $\bar{A}_i$  and  $h := h + 1$ , returns to Step 2;
  - 4: Apply the first element of the solution  $u_i^{\{h\}}(k|k)$  to the system;
  - 5: At the next time instant, measure the state of the current vehicle  $i$ , and communicate with its adjacent vehicles
  - 6: Set  $k = k + 1$ , go to Step 1;
- 

Table 1. Initial state of vehicles

| # vehicle | position | speed | acceleration |
|-----------|----------|-------|--------------|
| Vehicle 2 | 20       | 0     | 0            |
| Vehicle 3 | 12       | 0     | 0            |
| Vehicle 4 | 6        | 0     | 0            |

Table 2. Weights in the cost function

| weight | Q             | R | N  | $d_0$ | h |
|--------|---------------|---|----|-------|---|
| value  | diag(20,16,6) | 1 | 15 | 0     | 1 |

**Remark 4.** Since we are trying to control linear systems (6) and the involved constraint sets (8)(9)(10) are convex, the systems under control are inherent robust to small disturbances (Grimm et al., 2004; Yu et al., 2014).

## 4. SIMULATION RESULTS AND DISCUSSION

In this section, numerical simulations are conducted to illustrate the main results of this paper. We consider a platoon with four vehicles (one leader and three followers) under the predecessor-following topology, i.e., Fig. 1. Note that the acceleration of the preceding vehicle is treated as the reference.

The position and speed of the leader vehicle are  $s_1 = 30$  and  $v_1 = 0$ , respectively, and the acceleration of the leader vehicle is

$$a_1 = \begin{cases} 1.5 & t \in [0, 12s) \\ 1.5 - 0.1t & t \in [12, 27) \\ 0 & t \in [27, +\infty) \end{cases} \quad (16)$$

The initial states of the following vehicles are shown in Table 1, and the headway and the other parameters of Nash optimality based DMPC are shown in Table 2. For simplicity, neither relative speed between the two adjacent vehicles nor road comfort is considered in simulation.

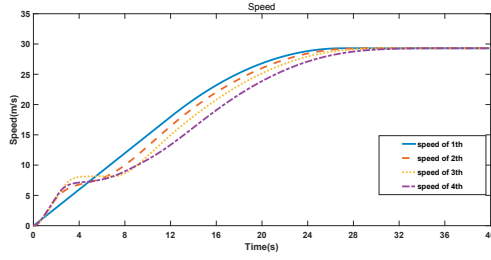


Figure 2. Speed with iteration

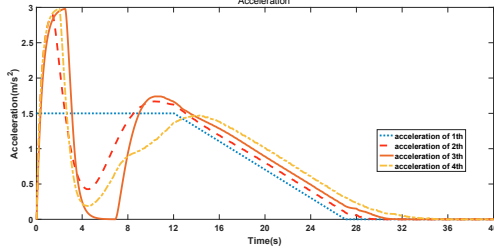


Figure 3. Acceleration with iteration

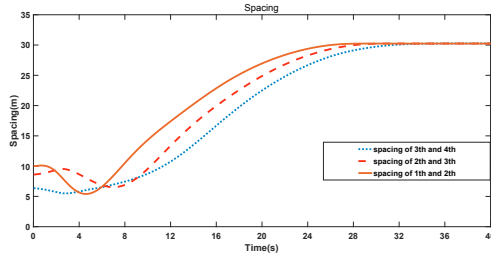


Figure 4. Spacing with iteration

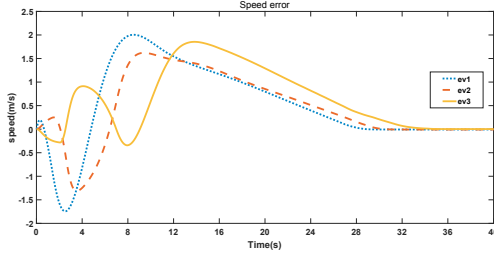


Figure 5. Speed error with iteration

Fig. 2-5 show the evolution of a platoon, i.e., the evolution of the speed, acceleration, spacing and speed error of the platoon under Nash optimality based DMPC.

Suppose that there is an unmeasurable disturbance acting on Vehicle 1,

$$\begin{cases} \dot{s}_1 = v_1 \\ \dot{v}_1 = a_1 \\ \dot{a}_1 = -\tau^{-1}a_1 + \tau^{-1}u_1 + w \end{cases}$$

with

$$w = \begin{cases} 0.2 & t \in [20, 20.2] \\ 0 & \text{otherwise} \end{cases}$$

Fig. 6-9 shows that, although there exist bounded additive disturbances, both collision avoidance and longitudinal oscillations elimination can be achieved. Thus, the vehicle platoon with the proposed scheme (Nash optimality based DMPC) has the potential to handle unexpected accident.

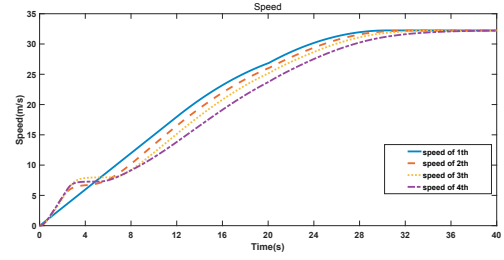


Figure 6. Speed with unknown disturbance

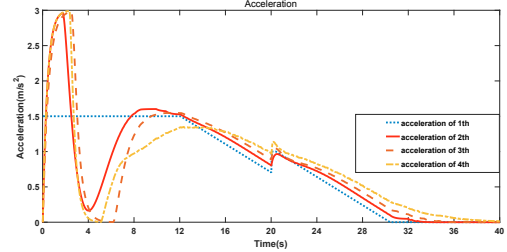


Figure 7. Acceleration with unknown disturbance

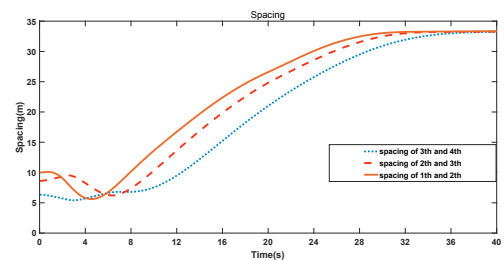


Figure 8. Spacing with unknown disturbance

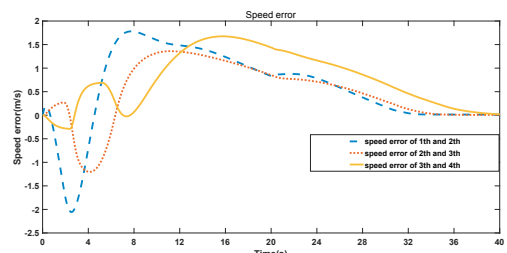


Figure 9. Speed error with unknown disturbance

## 5. CONCLUSION

In this paper, a distributed model predictive control based on Nash optimality was addressed for vehicle platoons, which can achieve a better performance through information exchange (communication) during the process of optimization. Acceleration of the preceding vehicle was used as kind of “reference” in the controller design, which might result in faster response and shorter inter-vehicle distance. Safety consideration was formulated as the time domain constraints in the individual optimization problem. Simulation results showed that the platoon can achieve good tracking performance with safety constraints satisfaction. Note that updates occur in parallel within a common global clock period in the implementation of DMPC, but tasks within the period do not need to be synchronized.

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